FUNDAMENTALS OF ELECTRICITY AND MAGNETISM

Instructor's Manual to Accompany

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INSTRUCTOR'S MANUAL TO ACCOMPANY

FUNDAMENTALS OF ELECTRICITY AND MAGNETISM

INTERNATIONAL STUDENT EDITION

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INTRODUCTION

The aim of this book is to provide a solid background for more advanced courses for those students who will continue in physics, while at the same time giving adequate coverage of the phenomena and theory of electricity at a meaningful level for those students who will go no further into the subject. On the basis of three years experimentation with the text, while in preliminary form, with students representing a broad spectrum of abilities, some success can be claimed. The major objectives of the course have been fulfilled for both the unusually gifted and motivated students and for those of more ordinary abilities.

The course for which this book was planned is designed for one semester and is usually taken by beginning sophomores. Approximately 39 lectures of 50 minutes each (or 26 lectures of 75 minutes each) have been available. Lecture demonstrations have been used to illustrate and emphasize the basic phenomena of electricity and magnetism. In addition, 12 laboratory sessions (one per week), lasting 2 to 3 hours have been given. Since the lecture sections have been large (up to 200), informal discussion sections (about 20 students) of one hour each week, often led by teaching assistants, have been found useful. Three lecture sessions have been used for midterm examinations, and a problem set has been assigned each week.

The students in this course have usually had a full year of college mathematics, including analytic geometry and have some understanding of both differential and integral calculus. They have also had a one-semester physics course in mechanics. Experience has shown that although students may have reasonable facility with the formalism of differentiation and integration, they may be weak in the ability to apply these techniques to problems in physics. For this reason, care has been taken to show with illustrative examples the relationship between physical problems and their mathematical solution. It is assumed that students are familiar with vector quantities, but the concepts are reinforced by discussions of unit vectors and scalar and vector products.

One of the most powerful techniques for the study of the vector quantities of electricity, as exemplified by the development of Gauss' law (Chapter 2), is the relationship between volume and surface integrals (Section 6.4). The use of these techniques in this

course for the study of electric and magnetic quantities simplifies the understanding of physics, and also helps the student to realize the importance and meaning of the mathematical concepts.

Instructors who are planning to give a course of this type may find helpful the following schedule of lectures, laboratory experiments, and answers to even-numbered problems.

Suggested Lecture Schedule

39 Lectures (50 minutes each)

Lecture	Chapter	Subjects
1	1	Coulomb's law
2	2	Electric fields, conductors, lines of force
3	2	Gauss' law, field calculations
4	3	Electric potential
5	3	Potential difference
6	3	Field vs. potential
7	4	Capacitances
8	4	Combinations of capacitors, stored energy
9	4	Force via change in stored energy
10	5	Dielectrics, polarization
11	5	Electric displacement, D
12	5	Stored energy, depolarization factor
13	6	Force between currents
14	6	Magnetic induction field
15	6	Magnetic flux, force on moving charges
16	7	Current, resistance
17	7	Current and voltage measurement, emf
18	7	Circuits, resistivity of metals, contact potential
19	8	Motional emf, Faraday's law
20	8	Lenz's principle, mutual and self inductance
21	8	Stored magnetic energy, ballistic galvanometer
22	9	Magnetic effects in matter
23	9	B vs. H, stored energy
24	9	Dia-, para-, and ferromagnetism
25	9	Permanent magnets, magnetic circuits
26	10	A-c circuits, L, C, R
27	10	Transients, filters
28	10	Generators, motors
29	11	Vacuum tubes
30	11	Transistors
31	12	Displacement currents
32	12	Development of Maxwell's equations

Lecture	Chapter	Subjects
33	12	Poynting vector, waves in guides
34	13	Gas discharge
35	13	Motion of charged particles in fields
36	13	Magnetohydrodynamic waves
37	14	Electric and magnetic quantum effects
38	14	Electric and magnetic quantum effects
39	15	Units and review

Comments on Lecture Schedule

Since different classes may well cover the material at different rates, it may be useful to indicate certain sections of the book which could be eliminated from the course, should time for all of the material not be available. In particular, the material in Chapter 13 could be omitted without destroying the unity of the course. This material on conduction in gases and magnetohydrodynamics has been included because of its foundations in electromagnetic theory, and because of current active research development, but it could clearly be postponed to a later course. Similarly, Chapter 14 on quantum effects could be omitted, especially when students will have further course work on modern physics.

If additional curtailment is necessary, parts of Chapter 9, on magnetism in matter, could be omitted, such as Sections 9.8, 9.9, 9.10, 9.11, and 9.12 on paramagnetism, diamagnetism, and ferromagnetism. These subjects involve the application of electromagnetic theory to solid state and, while of great interest and importance, could be omitted without serious disruption to the study of general electromagnetic theory. The same argument could be applied to Sections 7.10, 7.12, on resistivity in metals and contact potentials, as well as to the material on vacuum tubes, transistors, and klystrons in Chapter 11. However, for those students for whom this course is a terminal one in electricity, it would seem a serious omission to leave out all of these interesting applications of electromagnetic theory.

Laboratory Experiments

Laboratory experiments at the lower division level vary considerably from one college to another and depend largely on facilities and equipment available. A listing of experiments currently in use at Berkeley is given below. There is a wide range of choice in this matter and the author knows of no very strong arguments for one experiment over another. Much more important is the way in which

the laboratory is run and the degree of initiative which the students are called on to use.

Typical Experiments in Electricity and Magnetism

1) Equipotential Lines and Electric Fields

A mapping experiment using an electrolytic tray. Argument is made to show relationship of potential distribution in a two dimensional electrolytic tank containing electrodes to the potential distribution in a vacuum resulting from charged three dimensional electrodes. A probe connected to a potentiometer is used to plot equipotential lines and from these electric field lines are constructed. Experimental results are compared to theory for a right circular cylinder electrode with axis perpendicular to a uniform field.

2) Electrostatic Forces

Two parallel plane electrodes are charged to a known potential difference. The electrostatic force between them is measured by tilting the assembly until the component of the gravitational force on one of the electrodes causes it to fall away from the parallel position. Experimentally determined quantities allow evaluation of the electrostatic force constant in terms of ϵ_0.

3) The Potentiometer

A simple slide wire potentiometer is calibrated by means of a standard cell. The potentiometer is used to measure with precision the emf of a dry cell, and by inserting resistance loads across the dry cell, its internal resistance is measured.

4) Resistance Measurements, Wheatstone Bridge

a) An ohmeter circuit is studied and used to measure a series of unknown resistors.

b) A Wheatstone bridge circuit is used for further resistance measurements, including the resistance of a germanium diode. It is shown that the diode is non-ohmic.

5) Moving Coil Galvanometer

A moving coil galvanometer is calibrated by the student and used to measure a high resistance.

6) The Ballistic Galvanometer

The student studies the use of a ballistic galvanometer, learns how to take into account the damping factor of the instrument, and measures a capacity and a mutual inductance.

7) Magnetic Field Measurements

A magnetometer is used to compare the field of a permanent bar magnet with the earth's field. A test is made of the $1/r^3$ dependence of the field of the dipole.

8) The Vacuum Diode

Diode characteristics are studied. Effects discussed include thermionic emission, space charge effects, current-voltage characteristics. With the aid of an oscilloscope, the diode characteristics are measured.

9) The Capacity Bridge

A capacity bridge is used to measure the capacity of a capacitor and, using a parallel plate capacitor, the dielectric constant of several materials is obtained.

10) L, C, and R in A-C Circuits

The current-voltage and phase relations in a series LCR circuit are studied, using an oscilloscope.

11) The Vacuum Triode

The characteristics of a triode are investigated. Parameters defined, discussed, and measured include the amplification factor, mutual conductance, and plate resistance. A simple d-c amplifier circuit is constructed and its properties studied.

12) Magnetization and Hysteresis in Iron

The B-H curve and hysteresis loop of an iron bar is measured, using a ballistic galvanometer.

Films available

The following list of films represents a selection from the many now available on subjects involving electricity and magnetism. There may be some occasions on which instructors would like to use some of these during regular class meetings. In addition, a selection of these films could provide a stimulating series for a few voluntary or optional afternoon or evening meetings of the class. Since some of the films are quite short, several films could be shown at one meeting. A few of the films listed cover basic ideas of electricity but most of them give applications of electrical laws in various applied fields.

ANSWERS TO EVEN-NUMBERED PROBLEMS

Even Problems

1.2 $F = \frac{\sqrt{3}}{4\pi\epsilon_0} \frac{Q^2}{.01}$ newtons

1.4 a) $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{a^2}$ newtons

b) $W = \frac{1}{4\pi\epsilon_0} Q_1 Q_2 \left(\frac{1}{b} - \frac{1}{a}\right)$ joules

1.6 a) $q = -Q/2\sqrt{2}$ coulombs
b) No
1.8 a) 6.65×10^{15} rev/sec
b) 10.4×10^{-35} kg-m^2 rad sec^{-1}
c) 1.36×10^{27} m
2.2 a) Q/ϵ_0 lines
b) $N = Q/32\pi\epsilon_0$ lines
c) 1 line/m^2
2.4 Any spherically symmetric distribution
2.6 1.89×10^{11} Q newtons/coulomb
2.8 $E = (1/4\pi\epsilon_0)(Q/.4\sqrt{2})$ newtons/coulomb
2.10 $(40\, Q/4\pi\epsilon_0)$ joules
2.12 a) $\tau = E\, a\, Q \sin\theta$ newton meters
b) $W = 2a\, Q\, E$ joules
c) $T = 2\pi\, (I/a\, Q\, E)^{\frac{1}{2}}$ sec
2.14 $E = (12\, q\, a^2/4\pi\epsilon_0 r^4)$ newtons/coulomb
2.16 $A = 3Q/\pi R^4$
$E_{in} = (12Q/4\pi\epsilon_0 R^4)[\frac{1}{3} Rr - \frac{1}{4} r^2]$ newtons/coulomb
$E_{out} = (Q/4\pi\epsilon_0 r^2)$ newtons/coulomb
3.2 Plane $\perp \ell$ at midpoint. $V = 0$
3.4 $W = [qQ(r_A - r_B)/4\pi\epsilon_0 r_A r_B]$ joules
3.6 $V = (Q/4\pi\epsilon_0 R)$ volts. $W = (Q^2/4\pi\epsilon_0 2R)$ joules
3.8 $v = (2eV/m)^{\frac{1}{2}} = 0.59 \times 10^7$ m/sec
$W = 1.6 \times 10^{-17}$ joule $= 1.6 \times 10^{-10}$ ergs $= 100$ ev
3.10 [Answer not to be given]
3.12 $V_1 = (2d^2 V/b^2)$ volts
3.14 $F = (q^2/4\pi\epsilon_0 4a^2)$ newtons

3.16 $E_x = (bx - a)(x^2 + y^2)^{-\frac{3}{2}} + 3\,ax^2(x^2 + y^2)^{-\frac{5}{2}}$ volts/meter

$E_y = 3\,axy(x^2 - y^2)^{-\frac{5}{2}} + by\,(x^2 + y^2)^{-\frac{3}{2}}$ volts/meter

4.2 $C = 4\pi\epsilon_0 \Big/ \left[\left(\frac{1}{a} + \frac{1}{c}\right) - \left(\frac{1}{b} + \frac{1}{d}\right)\right]$ farads

4.4 $C = \dfrac{[C_1C_3C_4/(C_3 + C_4)] + C_1C_2}{C_1 + C_2 + [C_3C_4/(C_3 + C_4)]}$ farads

4.6 $V_2 = 2V_1$, $U_2 = 2U_1$

4.8 $C_1/C_2 = C_3/C_4$

4.10 $V_e = 5.56 \times 10^{-12}$ volt

$V_p = 1.02 \times 10^{-8}$ volt

4.12 $C = 0.71 \times 10^{-4}$ farad

4.14 a) $U = 10^{-2}$ joule

b) $U = 5 \times 10^{-3}$ joule

4.16 $a = (e^2/8\pi\epsilon_0 mc^2) = 1.28 \times 10^{-9}$ meter

4.18 a) $U = 8.85 \times 10^{-5}$ joule

b) $W = 1.77 \times 10^{-5}$ joule

c) $V = 1600$ volts

5.2 [Answer not to be given]

5.4 $U_2 = KU_1$ joules

5.6 $C_b/C_a = 4K_1K_2/(K_1 + K_2)^2$

5.8 $dQ_p = 2\pi r^2 P \sin\theta \cos\theta \, d\theta$

$Q_p^{\circ} = \pi r^2 P$ coulomb

5.10 $K = 1.5$, $\chi = 0.5$

6.2 $B = \mu_0 I\, 4\sqrt{2}/4\pi a = 1.13 \times 10^{-4}$ weber/m^2

6.4 [Answer not to be given]

6.6 $B = (\mu_0 I/2\pi b)\ln[(b + a)/a]$ weber/m^2

6.8 $r = (\mu_0 I_1 I_2/\sqrt{2}\,\pi mg)$ meters

6.10 $B = (\mu_0 \sigma \omega a/2)$ webers/m^2

6.12 $F = (\mu_0 N^2 I^2 a/b)$ newtons

6.14 a) $B = 1.82 \times 10^{-4}$ weber/m^2

b) $B = 1.96 \times 10^{-2}$ weber/m^2

6.16 $F = 3.33 \times 10^{-5}$ newton

6.18 $R_H = 0$

6.20 $B = .02$ weber/m^2 = 200 gauss

6.22 $v = 2.46 \times 10^7$ m/sec

6.24 $T = I\,B\,a$ newtons

6.26 $F = NI_0 2\pi r B_0 \cos\theta$ newtons, upwards

6.28 $B = (\mu_0 NI/4r)$ webers/m^2

6.30 $B = 3.58 \times 10^{-3}$ weber/m^2 = 35.8 gauss

7.2 Hall effect

7.4 $\tau = 2.09 \times 10^{-13}$ sec

7.6 $R = \frac{3}{4}$ ohm

7.8 a) $R = 8\frac{1}{3}$ ohms
b) $P_1 = 2.88$ watts
$P_2 = 7.20$ watts
$P_3 = 0.854$ watt
$P_4 = 0.64$ watt
$P_5 = 0.426$ watt

7.10 a) $R' = (R_1R + 2R_1R_2 + RR_2)/(2R + R_1 + R_2)$
b) $R' = 2\frac{4}{9}$ ohms
c) $R'' = 3$ ohms

7.12 $W = QV = QIR$ joules
$P = I^2R$ joules/sec = watts
$H = I^2R/4.18$ calories

7.14 a) $R_i = 0.05$ ohm
b) $R_i = 0.15$ ohm

7.16 $I_1 = \frac{11}{13}$ amp
$I_2 = \frac{2}{13}$ amp
$I_3 = \frac{9}{13}$ amp

7.18 $R_1 = 2985$ ohms, $R_2 = 12{,}000$ ohms, $R_3 = 135{,}000$ ohms
$R_{3v} = 3000$ ohms, $R_{15v} = 15{,}000$ ohms, $R_{150v} = 150{,}000$ ohms

7.20 $R_i = R_L$

8.2 $\mathcal{E}_{max} = \omega BnA$ volts

8.4 $M = \frac{1}{2}\pi a^2b^2/\ell^3$ henrys

8.6 [No answer to be given]

8.8 $U = 250$ joules
$\mathcal{E} = 900$ volts

8.10 $W/\text{cycle} = 2\pi R^2 e \dfrac{dB_r}{dt}$ joules/cycle

8.12 [No answer to be given]

8.14 $di/dt = 25$ amps/sec

8.16 $U = \frac{1}{3} \times 10^{-2}$ joule

8.18 $v = mgR/B^2\ell^2$ m/sec
$v' = 4v$ m/sec

9.2 [No answer to be given]

9.4 $mg = (2\mu_0 q_m{}^2/4\pi)\left[\dfrac{1}{x^2} - \dfrac{x}{(x^2 + L^2)^{3/2}}\right]$

9.6 $N_I = 8.95 \times 10^3$ amp-turns
$B = 0.5$ weber/m^2

9.8 Power density = 12 watts/m^3, $P = 24 \times 10^{-5}$ watts
$i_{max} = 0.04$ amp
$L = 2.5 \times 10^{-3}$ henry
L would decrease

9.10 $x = 3 \times 10^{-4}$ m

10.2 a) $I = \left\{V_0/[R^2 + (1/\omega C)^2]^{\frac{1}{2}}\right\} \sin(\omega t + \phi)$, $\phi = \tan^{-1} \dfrac{1}{\omega CR}$

b) $\Delta\Phi = 0$
c) $\Delta\Phi = 90^\circ$ (V_R leads V_c)
d) $V_R = V_0R/[R^2 + (1/\omega C)^2]$ volts
$V_c = (V_0/\omega C)/[R^2 + (1/\omega C)^2]$ volts
e) [No answer to be given]

10.4 a) $N\Phi = NBA \sin\theta$ webers
b) $\mathcal{E} = -N\omega BA \cos\omega t$ volts
c) $\mathcal{E} = N\omega BA$
d) $i = -N\omega BA \cos\omega t/(R^2 + \omega^2 L^2)^{\frac{1}{2}}$ amps
e) $\Phi = \tan^{-1}(\omega L/R)$
f) $\Phi = 0$
g) I_L lags V_L by 90°

10.6 $I_0 = 0.239$ amp
$I_{av} = 0$
$I_{rms} = 0.169$ amp
$P = 0$

10.8 [Answer not to be given]

10.10 a) $\omega_{max} = 1/(LC)^{\frac{1}{2}}$ rad/sec
b) $I_{max} = V_0/R$ amps
c) $\omega = \dfrac{\sqrt{3}\ R \pm (3R^2 + 4L/C)^{\frac{1}{2}}}{2L}$ rad/sec
d) $\omega = 0, \infty$

10.12 a) [No answer to be given]
b) $t = (L/R) \ln 2$ sec
c) $\tau = L/R$ sec

10.14 a) $t = (1/RC) \ln 2$ sec
b) $i_{max} = V_0/R$ amps, at $t = 0$
c) $q_{max} = C\ V_0$ coulombs at $t = \infty$
d) $t = (1/RC) \ln 2$ sec

10.16 $RC = 6.89 \times 10^{-4}$ sec

11.2 [No answer to be given]

11.4 [No answer to be given]

12.2 [No answer to be given]

12.4 a) 3×10^5 cycles/sec
b) 3×10^8 cycles/sec
c) 10^{10} cycles/sec
d) 3×10^{12} cycles/sec
e) 6×10^{14} cycles/sec
f) 3×10^{19} cycles/sec
g) 3×10^{20} cycles/sec

12.6 [No answer to be given]

12.8 $f = 1.5 \times 10^{10}$ cycles/sec

12.10 $i_D = 0.11$ ma

VISUAL BIBLIOGRAPHY

The Cathode Ray Tube—How It Works. 15 min, (1943). USN, MN-2104-a; United World Films.

Coaxial and Microwave Miracles. 11 min, (1952). Bell System.

Coulomb's Law. 31 min, (1958). PSSC.

The Diode. 17 min, (1945). Guide, USOE, United World Films.

Electric Fields. (1958). PSSC.

Electric Lines of Force. (1958). PSSC.

Electrostatics. (1958). PSSC.

In All Weathers. 26 min, (1950). Radar. British Information Services.

Magnet Laboratory. 25 min, (1958). PSSC.

Microwave Oscillators. 18 min, (1949). USA, TF 11-1567; United World Films.

Mirror in the Sky (Appleton and the Ionosphere). 21 min, (1958). EFVA, British Information Services, McGraw-Hill.

Photo-Emission. 18 min, (1961). McGraw-Hill.

Piercing the Unknown (Electronic Computers). 22 min, color, (1954). IBM, Modern.

Principles of the Transistor. 21 min, (1959). McGraw-Hill.

Propagation (Antenna Fundamentals). 13 min, color, (1961). McGraw-Hill.

Radio Antennas: Creation and Behavior of Radio Waves. 12 min, (1942). USAF, TF 1-474; United World Films.

Radio Receivers: Principles of Radio Receivers. 17 min, (1945). USAF, TF 1-474; United World Films.

Rotating Magnetic Fields. 13 min, (1945). Guide. USOE, United World Films.

The Story of Television. 24 min, b&w and color, (1956). RCA and Institute of Visual Training.

Television Receivers. 7 min, (1953). McGraw-Hill.

Television System. 14 min, (1953). McGraw-Hill.

The Transistor. 11 min, (1953). Bell System.

Travelling Electrical Waves. 50 min, si, (1936). D-c waves on open line, short circuited, and loaded lines. Massachusetts Institute of Technology.

The Triode: Amplification. 14 min, (1945). Guide. USOE, United World Films.

Vacuum Practice. 15 min, (1959). McGraw-Hill.
Vacuum Tubes. 11 min, (1943). Encyclopaedia Britannica Films.
Vacuum Tubes: Electron Theory and the Diode Tube. 16 min, (1945). USAF, TF 1-470; United World Films.

EXAMINATIONS

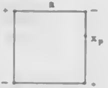
The following sample examinations show the level at which students may be expected to perform. The three midterm examinations cover successive parts of the course and are planned for 80-minute periods. One or more problems would be omitted for 50-minute examinations. The two final examinations given are planned for a three-hour period.

FIRST MIDTERM EXAMINATION, 80 minutes

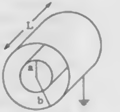
CLOSED BOOK EXAM

NOTE: Give units of all answers!

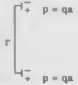
1. (20 points)
   Four charges of equal magnitude and of signs as shown are placed at the corners of a square of side a. Find the potential of a point p on the square, a distance x from a negative charge at the corner. For what value of x is V = o? Show in a sketch the locus of other points in the plane for which V = o. If there is a point or points in the plane at which E = o, identify it or them.



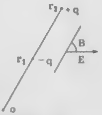
2. (20 points)
   Two coaxial metal cylinders of length L have radii a and b as shown. A total charge of +Q is placed on the inner cylinder. Use Gauss' law to find the field at a radius r between the two cylinders. Use this result to obtain the potential difference between the two cylinders.



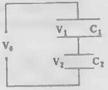
3. (20 points)
   Two dipoles of electric dipole moment p = qa lie along the same axis as shown. For $r \gg a$, find the approximate expression for the force between the dipoles in terms of p and r.



4. (20 points)
Charges $+q$ and $-q$ are located on a rigid rod which is free to rotate about the point o. The distances of the charges from o are r_1 and r_2 as shown. The rod makes an angle θ with a uniform field E. Find the torque acting on the rod about the point o. Express the answer in terms of the dipole moment p produced by the pair of charges. Show the direction of the torque. What is the net electrical force on the rod?



5. (20 points)
Two capacitors of known capacity C_1 and C_2 originally uncharged are connected to a source of known potential difference, V_0.



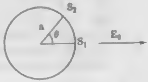
a. Find the expression, in terms of known quantities, for the potential difference V_2.

b. Suppose that the two charged capacitors are now isolated from the voltage source V_0, and that a third uncharged capacitor of known capacity C_3 is placed in parallel with C_2. Find the expression for the new potential difference across C_2.

SECOND MIDTERM EXAMINATION, 80 minutes

CLOSED BOOK EXAM

1. (20 points)
A spherical block of dielectric is placed in an originally uniform electric field E_0 as shown.
Given the values of the following quantities:



E_0 = original uniform field
χ = susceptibility of dielectric
ϵ_0 = permittivity of free space
E_{dep} = depolarizing field of induced surface charges

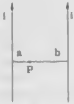
a. Write the expression for the dipole moment/unit volume, P, induced in the dielectric, in terms of the given quantities.
b. Write the expression for the induced surface charge density, σ_p, at the points s_1 and s_2.
c. Give the sign of the surface charge density at s_1 and s_2.

d. Find the dipole moment of the sphere, given the value of P.
e. Write the expression for the dielectric constant, K, in terms of given quantity or quantities.

2. (16 points)

Use Gauss' law for D, the electric displacement, to find E_1 and E_2 in the capacitor shown. Show in a sketch the Gaussian surface you use. The two dielectric blocks completely fill the space between capacitor plates. Find the potential difference, V, between the plates for a charge density σ_f on the plates. Consider k_1, k_2, a, b, and ϵ_0 known and express your answers in these terms.

3. (16 points)



Two long parallel wires carry a current i in the same direction. A point P lies in the plane of the wires, a distance a from one wire and b from the other. Find the magnetic induction field B at the point P (magnitude and direction). Suggestion: use Ampere's circuital law to calculate the contribution from each current.

4. (16 points)
Given that a particle of charge e and mass m is moving in a plane perpendicular to a uniform magnetic induction field B, and that it moves in a circular path as a result of the magnetic force, find the angular velocity ω with which it rotates.

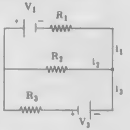
5. (16 points)

A wire of circular cross section, radius a, carries a uniform current I out of the paper. Find the magnetic induction B at a distance r from the center of the wire. $(r < a)$

6. (16 points)
Show from Gauss' law for E that if $\int_{cs} \underline{P}\cdot\underline{dS} = -q_p$, then it follows that $\int_{cs} \underline{D}\cdot\underline{dS} = q_f$.

THIRD MIDTERM EXAMINATION, 80 minutes

1. (20 points)
In the circuit shown, all R's have resistances of 1 ohm. V_1 and V_2 are each 1 volt. Using Kirchhoff's rules, find i_1, i_2, and i_3 (magnitude and direction).



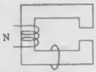
2. (20 points)
A conducting rod of length L rotates about one of its ends at an angular velocity ω radians/sec in a plane perpendicular to a uniform field B webers/m^2. Find the emf ϵ developed between the two ends of the rod:
   a. By the use of motional emf. [Start with $\underline{F} = q(\underline{V} \times \underline{B})$]
   b. By applying Faraday's law of induction. (Consider the rate at which magnetic flux is cut by the moving rod.)

3. (20 points)

A long thin paramagnetic rod placed in a uniform field B_0 is found to be magnetized to a value M in a region near the center of the rod.
   a. Find the value of H inside the rod near its center.
   b. Find the value of B in the same region in the rod in terms of B_0 and χ_m, the susceptibility.
   c. If the rod is replaced by a sphere of radius r, made of the same material, find the values of the following quantities inside the sphere: (The demagnetizing factor of a sphere is $L = \frac{1}{3}$.)
      1. The demagnetizing field.
      2. B in terms of H_0 and M.
      3. The dipole moment of the sphere (as seen from outside the sphere).

4. (20 points)
Two coils, 1 and 2, are placed near each other.
   a. Using the fact that the mutual inductance $M_{12} = M_{21}$, show that if half the flux ϕ_1 due to current in coil 1 links coil 2, then half the flux ϕ_2 in coil 2 links coil 1.
   b. Under these conditions, determine the mutual inductance M in terms of the self inductances L_1 and L_2.

5. (20 points)
A soft iron magnet of cross section A meters^2 has a flux path of length a meters in the iron and an air gap of b meter. The permeability of the iron (considered constant) is μ. A coil of N turns is wrapped closely around the magnet as shown. Assume all of the flux is confined to the iron and air gap path.

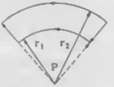


a. Determine the self inductance of the coil.
b. Suppose a superconducting winding is placed around the iron as shown. (This will be completely lossless, having zero resistance.) What now would be the self inductance of the original N turn coil?

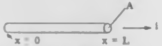
FINAL EXAMINATION, 3 hours

1. (20 points) An electrostatic field is in the x direction and is given by $E = kx$, where k is a constant. An electron, mass m and charge $-e$, is free to move when released from rest at a position $x = x_1$. Find the velocity of the electron when it reaches the position $x = 0$.

2. (25 points) A closed wire loop is made up of two circular quadrants of radii r_1 and r_2 about a common center and two radial connecting lengths as shown. When a current i flows clockwise in this loop, find the magnetic induction field B (magnitude and direction) at the center of curvature P in the plane of the loop.

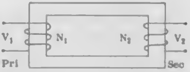


3. (25 points) The resistivity of a wire of uniform cross section A and length L varies along its length according to the equation $\rho = \rho_0 x^2$, where $x = 0$ at one end and $x = L$ at the other end. When a voltage V is applied between the two ends, find the current in the wire.

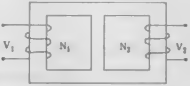


4. (25 points) A coil carrying 2 amps suffers a maximum torque of 0.3 newton meters in a fixed field. When the current source is removed from the coil and it is rotated with its axis perpendicular to the magnetic induction field at a frequency of 120 rev/min in the same field, what will be the maximum emf appearing across the terminals of the coil?

5. (25 points) Consider the ideal transformer shown, with N_1 and N_2 turns on the primary and secondary respectively. The peak voltage applied to the primary is V_1. The secondary peak voltage is V_2, and there is no load on the secondary coil.



   a. If the voltage source is sinusoidal, what is the rms primary voltage? (5 points)
   b. Derive the relationship between the peak voltages, V_1 and V_2, assuming the resistance of the primary coil is negligible. (10 points)
   c. If an iron bar is inserted as shown in the second figure, and if 60% of the flux caused by the primary goes through this bar rather than through the secondary coil, what will be the new relationship between V_1 and V_2? (10 points)



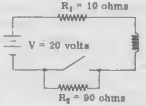
6. (25 points) A galvanometer which gives full scale deflection for 1 milliampere current and which has an internal resistance of 50 ohms is to be converted to an ammeter with full scale sensitivity of 15 milliamps. Show a diagram of how this can be done and find the value of the extra resistance which will be needed.

7. (30 points) A moving charged particle of mass m and charge e is to be caused to rotate at constant velocity in circular motion by either a magnetic induction field B or an electrostatic charge Q at the center of its circular path.
   a. Calculate for each case the angular frequency ω of the particle in terms of e, m, B, Q, r, $4\pi\epsilon$, as needed. (12 points each)
   b. For what radius will the angular frequency be the same for both cases? (6 points)

8. (30 points) a. Calculate by means of the energy per unit volume in an electric field the stored energy U in the space outside a metal sphere of radius r, when a charge Q is placed on the sphere. (12 points)
   b. Compare this with the stored energy as computed from the work necessary to charge a spherical capacitor to a total charge Q. (12 points) (Calculate the capacity of the sphere needed for this.) (6 points)

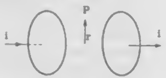
9. (30 points) You are to calculate the Larmor frequency of an electronic orbit in an atom resulting from an applied magnetic field B. Assume a circular orbit of constant radius r, with an original angular velocity ω_0. Take the orbit perpendicular to the applied field. Calculate the accelerating force tangential to the orbit on the electron while B is increasing from 0 to its final value. This force results in a calculable change in the momentum of the electron. From this, Δv or $\Delta\omega$ can be found. $\Delta\omega$ is the Larmor frequency.

10. (35 points) In the circuit shown the switch is originally closed and a steady current flows. At $t = 0$ the switch is opened, putting R_2 into the circuit.



   a. What is the steady current before the switch is opened? (2 points)
   b. Find the expression for di/dt at any time after the switch is opened. (8 points)
   c. Find the expression for the current at any time after the switch is opened. (8 points)
   d. What is the final steady current? (5 points)
   e. What is the maximum value of di/dt? (5 points)

11. (30 points) A constant current i is flowing into a parallel plate capacitor with circular plates of area A, plate separation L, and thereby charging it.



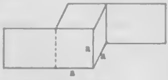
   a. Find dE/dt by first finding the rate of increase in surface charge density σ on the plates. (15 points)
   b. Use Maxwell's equation involving displacement current to find $\oint \mathbf{H} \cdot d\mathbf{l}$ in the space between the plates, and in particular find the value of H at P, at the edge of the region between the plates. Neglect fringing effects. (15 points)

FINAL EXAMINATION, 3 hours

Closed Book Examination: Maximum score = 300 points

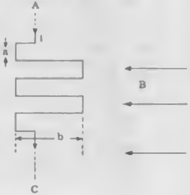
1. (15 points) The spacing between neighboring charges in the diagram is a. Each charge is $+e$ or $-e$. Find the potential at the point at the center. Find the field at the same point.

2. (15 points) A cube of material of resistivity ρ and of dimensions a is connected to strips of high conductivity metal as shown in the diagram. Compare the resistance of this cube with that of a cube with dimensions 2a, having the same resistivity.



3. (40 points) Consider a plane boundary between two dielectric media, having dielectric constants K_1 and K_2. Determine the boundary conditions on E and D. If the electric field makes an angle φ_1 with the normal to the boundary, find the value of the angle of refraction, φ_2 in terms of φ_1, K_1 and K_2.

4. (20 points) The figure shows a rigid conductor carrying a current i, free to rotate about the axis AC. A uniform magnetic field B is pointed from right to left. Find the torque on the conductor (magnitude and direction).

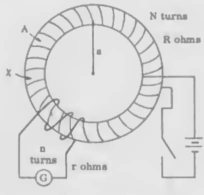


5. (40 points) A 60-cycle a-c voltage of 100 volts (rms) is connected to the load as shown. A current of 10 amperes (rms) passes through the load. A wattmeter placed in the circuit reads 707 watts. The a-c current is found to lead the voltage. Find the power factor. From the information given,

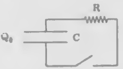
draw a circuit which corresponds to the load and determine the values of the circuit elements you use.

6. (30 points) A plane is traveling east from San Francisco to New York, at a speed of 300 meters/sec (about 600 miles/hr). Assume the magnetic field to be 1 gauss (1 weber/m^2 = 10,000 gauss), and that it is tipped downward at an angle of 60° from the horizontal. Find the emf between wing tips, separated by 40 meters.

7. (40 points) A paramagnetic material of magnetic susceptibility χ, in the form of a toroidal solenoid, is wrapped with N closely wound turns of wire which can be connected to a source of emf V volts. The resistance of the coil is R ohms. Another coil of n turns is wrapped around the solenoid and connected to a ballistic galvanometer. This secondary circuit has a resistance of r ohms. Find the magnetic induction flux φ in the solenoid when the emf is connected to the circuit. What is the value of j^s_{mag} when the emf is connected? What charge Q flows through the galvanometer when the emf is disconnected?



8. (40 points) A capacitor C, with an initial charge Q_0, is discharged through a resistance R. Find the charge q remaining on the capacitor after a time t. Compare the initial stored energy with the energy dissipated in the resistance during the entire time of discharge (from $t = 0$ to $t = \infty$). Show detailed calculations.



9. (20 points) Two identical coils are connected in series and spaced in such a way that half the flux from coil A goes through coil B. Given that the self inductance of coil A is L_1 henrys, determine the self inductance of the two coil combination, assuming connections such that the fluxes add (rather than subtract).

10. (40 points) A cylindrical resistance of length L, radius r, and resistance R is connected to a circuit which places a potential difference V between its ends. Find the magnitude and direction of $\underline{S}$, the Poynting vector, at all points on the surface of the resistance. Then evaluate $\int_{CS} \underline{S} \cdot \underline{dA}$ over the entire surface of the resistance, to show that the flow of energy as described by the Poynting vector accounts for the rate of energy dissipation in the resistor.